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Designing and optimization of new composite pallet

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Abstract

The original code KEDRO for design of experiments, analysis and multiobjective robust optimization is used for designing of a new composite pallet. Firstly code is tested for non-deterministic optimization of the two bar truss problem. Then robust optimization problem of composite pallet is solved. The FE-model of the composite pallet is considered and solved accurately as multi-ply shell structure. The fiber-reinforced polymer material mechanical properties and two main operation cases of the loaded pallet are taken into account during deterministic structural optimization procedure. Next, the same problem is considered as non-deterministic taking into account possible uncertainties of the pallet supporting conditions. In both cases shape is defined using CAD based NURBS curves. Appropriate shapes of the stiffness ribs are found for best performance of the structure. As a result of optimization, the competitive design of composite pallet is developed.

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Keywords: shape optimization; pallet; composite.

1. Background

Powerful methods for shape and topology optimization of mechanical engineering objects such as ground structure [1], homogenization [2] and solid isotropic material with penalization [3] are broadly used for industry problem solving. Popular new approaches for shape optimization are morphing, implicit parameterization and CAD-based direct parameterization [4-6]. Nevertheless real life problems are almost always non-deterministic. Uncertainties appear for example due to material, load and geometry fluctuations: manufacturing tolerances, model errors, changing environments and noisy measurements. Robust optimization approaches [7-10] seek to limit the effects in quality of the solutions due to uncertainties.

In recent years the CAD-based direct parameterization approaches [11] have become highly effective and popular due to rapid development of the integrated

CAD/CAE software systems and advanced metamodeling techniques [12-15]. NURBS utilization for the freeform curves representation of CAD models can give even more benefits for those techniques. Also references to the kriging based optimization methods [16,17] are commonly given to solve deterministic optimization problems. For non-deterministic optimization problems, such as composite structures that account for uncertainties, the optimization is usually based on double loop approaches where the uncertainty propagation is recursively performed inside the optimization iterations. Often the uncertainty estimation for the given point is based on a meta-model, thus allowing reduction of computational time but introducing additional bias in the estimates. In the work [18] a single loop kriging based method for minimizing the mean of an objective function is proposed: simulation points are calculated in order to simultaneously propagate uncertainties, i.e., estimate the mean objective function, and optimize this mean.

Software KEDRO was originally developed as a collection of non-gradient-based optimization tools in

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Machine and Mechanism Dynamics Research Lab of Riga Technical University. It includes methods and tools for statistical data sampling, approximation and metamodeling methods, and metamodel-based multiobjective optimization, including realization of method [18] that is necessary for non-deterministic optimization of the proposed pallet. KEDRO and CAE simulation code (mainly FE, CFD and multibody dynamics software) remain entirely independent, with data being transferred between KEDRO and the simulation code through writing and reading text files. KEDRO does not require access to the source code of the user's simulation software.

2. Case study: two bar truss

Here we demonstrate the use of KEDRO for non-deterministic optimization of the two-bar truss structure, which is a popular testing example for metamodeling, constrained optimization and robust optimization [19].

The two-bar test case aims at designing a structure made of two cylindrical bars joined together at the top as shown in Figure 1. A downwards force F is applied to the top of the structure.

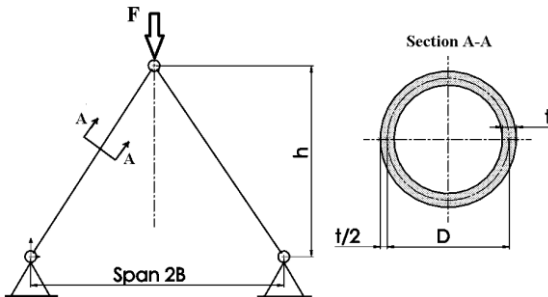


Fig. 1. Two-bar truss.

The design variables are the bar diameter D and structure height h . The objective is to minimize the total volume V . Buckling and strength failures give the formulation constraints:

$$\min_{D,h} V \text{ such that } \sigma \leq \sigma_{\max} \text{ and } \sigma \leq \sigma_{crit} \quad (1)$$

where σ is the stress on the bar, σ_{\max} is the yield strength, and σ_{crit} is the maximum buckling load. The two-bar problem is simple enough to have analytical expressions for the objective function and constraints:

$$V = 2\pi D t \sqrt{B^2 + h^2}, \quad \sigma = \frac{F \sqrt{B^2 + h^2}}{2\pi D h}, \quad \sigma_{crit} = \frac{\pi^2 E (D^2 + t^2)}{8(B^2 + h^2)} \quad (2)$$

For the robust optimization we have two design variables D and h , and additional four parameters – constants with fluctuations (unmapped analysis variables, see [20]). All parameter nominal values and their standard deviations are given in Table 1.

Table 1. Two-bar truss parameters. The fluctuations around the nominal values are Gaussian and all parameters are independent of each other.

Parameter	Nominal value	Std. Dev.
D bar diameter (mm)	20-80	1
t bar wall thickness (mm)	2.5	0.1
h structure height (mm)	278-936	3
B half structure width (mm)	750	5
E Young's modulus (N/mm ²)	210000	21000
F applied force (N)	150000	15000
σ_{\max} Yield strength (N/mm ²)	400	-----

To show the possibilities of metamodeling, we will use numerical experiments by calculating the response functions V , σ , σ_{crit} according to the design of experiments. We will use 6 input factors for metamodels. Table 2 shows the notation and limits of variation for all input factors.

Table 2. Two-bar truss parameters. The fluctuations around the nominal values are Gaussian and all parameters are independent of each other.

Factor = Parameter	Limits
x1=D	20-80
x2=t	2.4-2.6
x3=h	278-936
x4=B	745-755
x5=E	190000-230000
x6=F	120000-150000

111-point 6-factor MSE optimized design of experiments was used. The limits for “noisy constants” x_2 , x_4 , x_5 , x_6 were set \pm one standard deviation from nominal value. The responses are y_1 – stress σ , y_2 – buckling stress σ_{crit} and y_3 – volume V . The accuracy of approximation for tests with known response functions f_{test} was measured with the relative average prediction error σ_{test} in additional confirmation points not used in model building:

$$\sigma_{test} = 100\% \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N (f_{test}(z_i) - \hat{f}_{test}(z_i))^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^N (f_{test}(z_i) - \bar{f}_{test})^2}} = 100\% \sqrt{\frac{MeanSquareError}{Variance}} \quad (3)$$

where z_i – confirmation points ($i=1, \dots, N$), $\hat{f}_{test}(w_i)$ – approximated value of test function, \bar{f}_{test} – average value of test function in confirmation points. 100000 uniformly randomly selected confirmation points (Latin hypercube sample) were used in the region of interest.

The kriging approximation was used for all three responses (see Figure 2). The greatest approximation relative error was f or y_1 – 1.53%. The relative cross-validation 1.95%, in most cases the cross-validation error is pessimistic – the actual error is less than the cross-validation prediction. 1.53% is a good result; if the error would be measured relative to the full range of the change of the stress response (1087N/mm²), the relative error would be 0.28%.

TestSigma=1.52915901975475%

Functions Yr.	Strength	CrStrength	Volume
Method	Kriging	Kriging	Kriging
Sigma Cross	3.933738	1.758704	287.692699
Sigma Cross%	1.952612%	0.314628%	0.101197%
Sigma	0.000179	0.000002	0.022784
Sigma%	0.000089	0.000000	0.000008
Sigma0	0.000185	0.000003	0.023538
Sigma0%	0.000092	0.000000	0.000008
MeanExpValue	384.541103	801.853137	766800.103027
StDev of Exp	201.460314	558.979362	284291.025622
Exp. Range	1087.954072	2492.878252	1224913.739257
MaxError	-0.000694	-0.000009	0.065113
Bad Point No.	82	59	84
Max Rel Error	0.00%	0.00%	0.00%
BadRelPointNo.	82	61	84
No.ofActualExp	111	111	111

Fig. 2. Kriging approximation results (screenshot of KEDRO).

The solution of the deterministic problem is $D = 37.876$ mm, $h = 608.89$ mm, in which case $V = 0.5747631$ dm³, $\sigma = 399.99995$ N/mm², $\sigma_{crit} = 400.00019$ N/mm².

Table 3 shows the deterministic and robust optimization results for exact mathematical models and kriging metamodels. The optimization results obtained by the 111-run 6-factor experimental design are relatively close to the result obtained by using the exact mathematical model. It must be noted that the difference between the metamodel and the exact model is less than the standard deviation system parameters. The optimization method used was the reliability type optimization [19] and [20] and requires 95% probability of satisfaction both constraints (1).

Table 3. The results of two-bar truss optimization. Robust optimization with 95% confidence probability.

	Deterministic exact	Robust exact 95%	Deterministic metamodels	Robust Metamodels 95%
D	37.88	46.61	37.95	45.53
h	608.89	711.65	611.23	747
V	0.574763	0.75713	0.576746	0.757050
Worst case V	0.631675	0.82869	0.622671	0.815109
$\bar{\sigma}$	399.9999	297.65	399.99	481.14
5th and 95th percentile	235-567	194-400	-----	198-400
σ				
$\bar{\sigma}_{crit}$	400.00128	528	400	481
5th and 95th percentile	307-493	409-645	-----	398-568
σ_{crit}				

3. Pallet shape optimization under uncertainty

Nowadays competitive design of pallet, depending on area of application, should provide proper balance between six interactive characteristics: 1) strength; 2) stiffness; 3) durability; 4) functionality; 5) cost and 6) opportunities of recycling. Therefore common wooden pallets are often replaced with more effective designs: a) molded presswood pallets b) plastic pallets and c) composite pallets (e.g. fiber reinforced thermoset). New designs of pallets provide obvious benefits - superior strength and weight ratio, nestable design concept (see Figure 3b), increased service life, better corrosion and impact resistance, and many others. Presswood pallets become as main alternative to common wooden pallets and are widely used in many areas. The stiffness of such pallets is controlled by appropriate ribs topology and shapes (see Figure 3a).

The proposed new design of composite pallet (with dimensions 1200x800x160mm) (Figure 4a) could be a solution for modern automated distribution systems if its design is able to carry loads up to 19620 N during operation conditions with the factor of safety (FOS) of at least 2. For this reason fiber-reinforced polymer (FRP) [21] compression molding process [22] must be used for pallet manufacturing and also appropriate shapes of strengthening ribs 1-3 (Figure 4a) must be found for maximal performance of the structure.

At the beginning of the previously developed optimization loop [23] and [18], we need to minimize the number of required parameters to accurately specify shapes of strengthening ribs of the pallet. We want to consider only smooth shapes for stiffness ribs

that at same time are effective from the aspect of structural integrity.

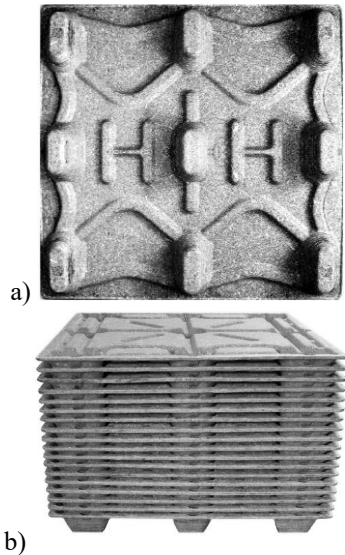


Fig. 3. Presswood pallets: a) curved stiffness ribs b) stackability.

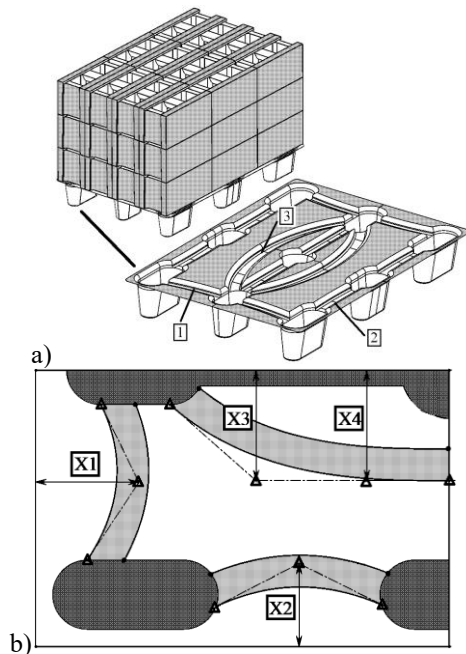


Fig. 4. 3D model a) of loaded pallet and its stiffness ribs 1-3 and b) NURBS parameterization of pallets stiffness ribs in $\frac{1}{4}$ pallet layout.

Therefore, due to the symmetry of the pallet the shape effective parameterization with 4 parameters ($X1$, $X2$, $X3$ and $X4$) is proposed, as shown in Figure 4b. The shape of each stiffness rib is controlled by control points of a non-uniform rational basis spline (NURBS) polygon. The shape is controlled using a

small number of parameters that is important for successful optimization, especially for the non-deterministic case.

The composite material of the pallet consists of two plies of FRP (Table 4), each has 3 mm thickness. The fibers of the second ply are orientated perpendicular to the fibers of the first ply (Figure 5) in XY plane that provides high strength in both X and Y directions. The strength of the pallet is defined with FOS using the Tsai-Wu criterion [24] which is best applied to orthotropic FRP that has unequal strength in tension and compression. The shell finite elements model of the composite pallet is considered and solved as a multi-ply structure (Figure 6), taking into account the symmetry of the pallet structure.

Table 4. Mechanical properties of FRP.

Elastic Modulus	$E_X = 40000$ MPa; $E_Y = 10000$ MPa; $E_Z = 10000$ MPa
Poisson's Ratio	$\nu_{XY} = 0.26$; $\nu_{YZ} = 0.25$; $\nu_{XZ} = 0.26$
Shear Modulus	$G_{XY} = 4500$ MPa; $G_{YZ} = 4000$ MPa; $G_{XZ} = 4500$ MPa
Mass Density	$\rho = 1900$ kg/m ³
Tensile Strength	$\sigma_{XT} = 1060$ MPa
Compressive Strength	$\sigma_{XC} = 600$ MPa
Shear strength	$\sigma_{XY} = 70$ MPa

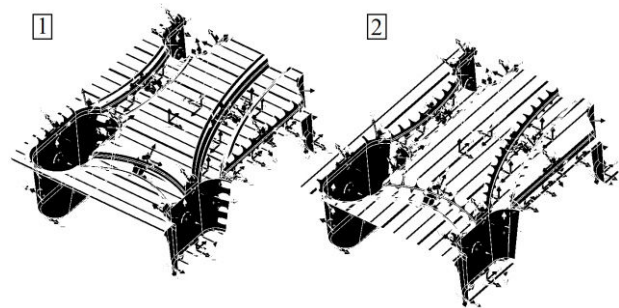


Fig. 5. Definition of fibers directions in 2 plies for $\frac{1}{4}$ of 3D model.

Two main operation cases of the loaded pallet are simulated: (a) the pallet stays on the rigid basis and (b) the pallet is transported on forks (Figure 6). In both operation cases the load is assumed to be deterministic and uniformly distributed and applied normally downward to the top surface of the pallet. The case (b) cannot be solved solely as deterministic problem due to uncertainties of supporting conditions (the distance between forks of the lifting equipment is

variable). Therefore in the case (b) the problem of the rib shape optimization must take into consideration the random uncertainty in the pallet supports. It is assumed to be symmetric but non-deterministic: supporting areas have constant width but different support placements – X_5 on the stiffness ribs as shown in Figures 6b & 7(I-II).

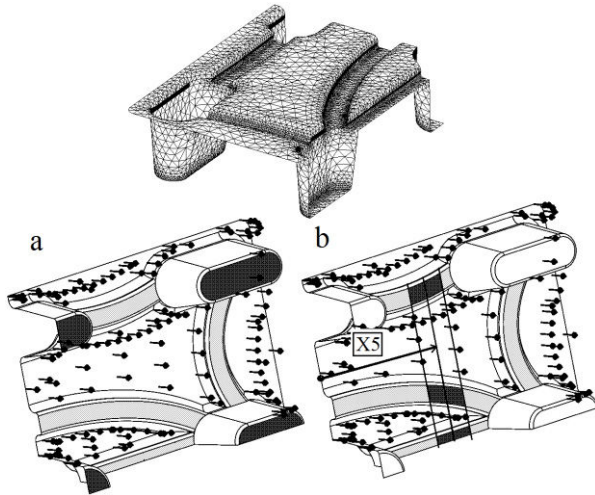


Fig. 6. The shell FE model of 1/4 pallet, and operation cases (a) & (b): arrows show load direction, supporting areas are colored black.

The results of finite elements analysis show obvious importance of the rib shapes for pallet structure stiffness and strength properties. Also a known fact is confirmed: pallet case (b) causes higher deflection and lower FOS. Therefore, only case (b) is considered during optimization.

Next, the deterministic shape optimization problem is defined:

$$\min_{X_n} \delta \quad \text{such that } FOS \geq 2 \text{ and } m \leq 4.688 \quad (4)$$

where X_n consist of factors X_1 - X_5 ; δ is pallet maximal deflection; and m is 1/4 pallet mass. In the first step X_5 is assumed taking 2 extreme values: the results are shown in Figure 7(I-II) and in Table 5 for each case. By moving supporting areas to the side of the pallet (increasing X_5), the maximal deflection of the pallet tends to relocate from the side of pallet to the center part (Figure 8) and at same time the level of deflections decreases.

In the next step robust optimization is performed. The uncertainty in X_5 is assumed uniform and is propagated using MC simulations on kriging metamodels. The constant number of MC simulations 10E4 is used which is further increased to 10E6 when higher MC accuracy is needed. The 96th percentile is

used for criterion and constraint satisfaction. Obtained stiffness ribs shapes and indices are shown on Figure 7(III) and in Table 5.

Table 5. The results of pallet shape optimization.

		Optimization cases (see Figure 7)		
	Range, mm	I	II	III
X1	95-220	182.6448	192.5575	219.827
X2	25-220	79.7038	62.8432	114.209
X3	135-195	135		
X4	160-195	160		
X5	200-270	200	270	160-270
		(fixed)	(fixed)	
FOS	metamodels	2	2.5	4.72*
	actual	2.2	2.53	
δ , mm	metamodels	3.6869	1.7313	4.719*
	actual	3.6806	1.7213	
m , kg		4.688		4.695*

* 96% percentile

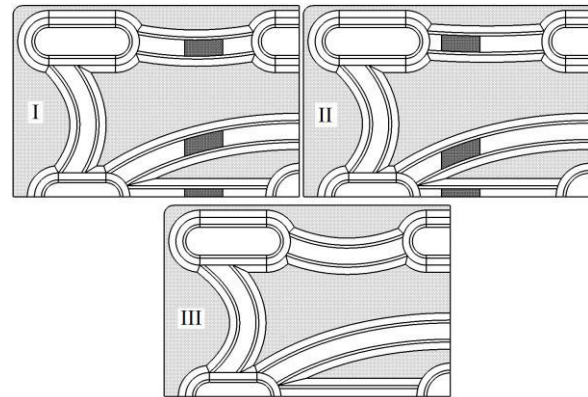


Fig. 7. The results of stiffness ribs shape optimization in 1/4 3D model of pallet for case (b): deterministic optimization for I) $X_5=200$ mm and II) $X_5=270$ mm; III) robust optimization (the fluctuations around the nominal X_5 values are uniform).

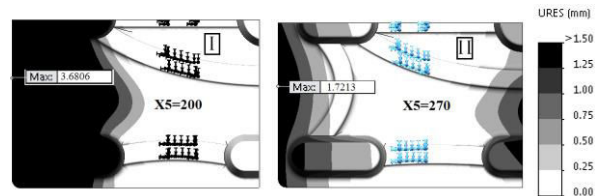


Fig. 8. The maximal deflection of the pallet for case (b): deterministic optimization I) $X_5=200$ mm, and II) $X_5=270$ mm.

4. Conclusions

The software KEDRO was successfully tested and used for robust optimization of composite pallet:

The test problem of two bar truss confirmed that it is possible to obtain a different solution considering structure optimization problem as non-deterministic. Solutions of deterministic and more actual robust optimization problem are compared. The new pallet design was found.

The pallet manufacturing technology and other causes of uncertainties in design, e.g., improper dimensions and material properties due to characteristics of compression molding could be defined and taking into account in the next stages of designing.

Acknowledgments

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