BRQ Business Research Quarterly (2014) **17**, 31–46

BUSINESS RESEARCH QUARTERLY

www.elsevier.es/brq

ARTICLE

Diluting the perverse element of rational altruism☆

Javier García-Bernal*, Marisa Ramírez-Alesón

Departamento de Dirección y Organización de Empresas, Universidad de Zaragoza, Spain

Received 19 April 2012; accepted 11 December 2012
Available online 20 March 2013

JEL CLASSIFICATION
D64;
M21;
D23

KEYWORDS
Asymmetric team members;
Rational altruism;
Second best optimal sharing rules;
Perverse Element of Rational Altruism

Abstract Some authors have demonstrated the value of egalitarian sharing rules in teams, even when team members have distinct abilities and make different contributions to team performance. However, we show the appearance of an undesirable component of rational altruism when marginal productivities differ across team members and an egalitarian sharing rule is used. We call this new component of rational altruism the Perverse Element of Rational Altruism (PERA). The presence of the PERA decreases team efficiency. In this sense, and considering several scenarios, an analysis of welfare implications and the evolution of team efficiency are presented.
© 2012 ACEDE. Published by Elsevier España, S.L. All rights reserved.

1. Introduction

The flattening of hierarchies and the implementation of team-based forms of work organization have been common practices among firms in recent years (Hamilton et al., 2003). The organization of work in teams increases group efficiency by taking advantage of the complementary resources and abilities of team members and also encourages the transfer of codified knowledge (Lazear and Shaw, 2007). However, as is well known, team-based organization raises unique motivation problems: Team production implies that the total output of the group cannot be broken down into the individual contribution of each of the team members. In addition, team organization generally implies self-management of work and no direct supervision.

☆ The authors are grateful for the financial support received from MICINN-FEDER through research Project ECO2009-09623 and from DGA-FSE through CREVALOR research group. Over the years we have received helpful and useful comments on a succession of drafts from Dr Vicente Salas-Fumás.
* Corresponding author.
E-mail addresses: jgbernal@unizar.es (J. García-Bernal), mramirez@unizar.es (M. Ramírez-Alesón).

2340-9436/$ – © 2012 ACEDE. Published by Elsevier España, S.L. All rights reserved.
http://dx.doi.org/10.1016/j.cede.2012.12.002

Therefore, since neither the individual marginal productivity contribution to the total output, nor the amount or quality of the effort given by each individual team member can be measured, the team's compensation has to be tied to the output of the group as a whole, creating inefficiency in the form of free riding and shirking behavior (Alchian and Demsetz, 1972; Holmström, 1982).

Holmström (1982) formally showed that there is no rule of sharing the joint output among team members that gives the first best welfare maximizing solution and satisfies the binding budget constraint at the same time. The lack of such a sharing rule does not preclude the interest in examining the characteristics of second best sharing rules that could be applied to induce effort in team production. Most existing theoretical studies on optimal motivation in team organizations assume symmetry among team members in the sense of an equal marginal productivity contribution by each team member, which could justify why most of the existing work on this topic uses equal sharing rules to explain team behavior.

However, there is some debate among scholars as to whether it is appropriate or not to use egalitarian sharing rules when team members have distinct abilities and make different contributions to team performance.

Studies of the disadvantages of equal sharing rules are numerous in the literature. For example, Wilson (1968) shows that equal sharing cannot be optimal in heterogeneous teams in which team members have different risk tolerances. Farrell and Scotchmer (1988) show that equal sharing will lead to inefficiently small teams if the team members differ in ability. Sherstyuk (1998) not only stated the weaknesses of equal sharing for teams with heterogeneous team members, but also showed that the equal sharing rule will induce team members with similar abilities to form partnerships thereby preventing the formation of mixed ability teams. Kräkel and Steiner (2001) show that equal sharing is not optimal, even when team members are completely homogeneous, due to the fundamental trade-off between incentives and risk sharing. Finally, Goerg et al. (2010) suggest that equal treatment of equals is neither a necessary nor sufficient prerequisite for eliciting high performance in teams.

However, in recent years, some authors have again established the convenience of applying egalitarian sharing rules. Specifically, Bose et al. (2010) demonstrate the value of ''equal pay'' policies in teams, even when team members have distinct abilities and make different contributions to team performance. Egalitarian sharing rules eliminate the incentive that each team member otherwise has to sabotage the activities of teammates in order to induce a more favorable reward structure. Bartling and Von Siemens (2010) show that with inequity adverse team members, which suffer disutility arising from differences between one's own payoff and others' payoffs (Fehr and Schmidt, 1999), the equal sharing rule is the only sharing rule that maximizes the team members' incentive to exert effort.

Our paper elaborates on this current debate and provides new arguments against the application of egalitarian sharing rules in teams with heterogeneous members. The starting point for this paper is the concept of rational altruism introduced by Rotemberg (1994) where each team member appears to be altruistic to the other parties even though his/her true interests are just selfish preferences. Based on this concept, we identify and assess a new and undesirable component of rational altruism which could appear when the application of egalitarian sharing rules result in decreases in team efficiency. We define efficiency as the maximization of social welfare (or wealth creation), which corresponds to the sum of the utilities of the various team members.

Rotemberg (1994) shows that, in the equilibrium of a two stage game, rational altruism implies more effort and greater wealth creation than could be expected from simple selfish behavior. However, our paper proves that the presence of rational altruism may not always be beneficial for the fostering of cooperation among team members. We find that when team member productivity differs, the use of equal sharing rules distorts the way which team members choose their degree of ''altruism''. Specifically, less productive team members have incentives to simulate altruistic feelings (by overplaying their level of effort) and increase their parameters of rational altruism, which corresponds to an attempt to expropriate rents from the more productive team members. As a consequence the more productive team members will anticipate this perverse behavior and decrease their degree of altruism and their effort. This induced change in the rational altruism parameter that is masked under the concept of rational altruism stipulated by Rotemberg (1994) is called the *Perverse Element of Rational Altruism* (PERA). In terms of wealth creation, our results show that the negative effect derived from lower levels of effort by the more productive team members is greater than positive effect derived from higher levels of effort by the less productive team members. Therefore, the presence of the PERA decreases team efficiency.

At this point, the potential PERA presence should be incorporated into the design of optimal sharing rules that could be applied to dilute the undesirable effects of the PERA. Specifically, this paper shows that the counterproductive effect of the PERA is not present if team members are paid based on their relative marginal productivities level.

The contribution of our paper is twofold. First, the paper provides new arguments against the use of egalitarian sharing rules in teams with heterogeneous members. Second, the paper identifies and measures the perverse effects of a component of rational altruism not previously identified in the literature (to the best of our knowledge): the Perverse Element of Rational Altruism (PERA).

The rest of the paper is organized as follows. We first focus on the design of second-best optimal sharing rules, both with and without rational altruism. In the third section, we show the sources of inefficiency that appear when using equal sharing rules in a context of heterogeneous team members and rational altruism. Part of this inefficiency is summarized by the concept of the Perverse Element of Rational Altruism (PERA). In the fourth section, we analyze, considering several scenarios, welfare implications and team efficiency evolution. Finally, we discuss the main conclusions and implications.

2. Second-best optimal sharing rules

The impossibility theorem of Holmström (1982) concerning the non-existence of a sharing rule that gives the first best level of output in a budget constrained team does not preclude the interest in examining the characteristics of second best sharing rules that could be applied to induce team production effort. Most of the existing theoretical papers on optimal motivation in team organizations assume symmetry among team members in the sense of the equal marginal productivity contribution of each team member, which could justify why most of the existing work on this topic uses equal sharing rules to explain team behavior. This is also the case in Rotemberg's work where, under the assumption that all members are symmetric in terms of their marginal productivity contribution to the team output, it is shown that the altruism parameter will also be the same for all team members (symmetric equilibrium).

However, a hypothesis of symmetric team members can be very unrealistic. Therefore, in this paper we examine the choice of second best output sharing rules in teams with asymmetric members, and how the choice of these rules is affected by the rational altruism of team members.

2.1. A proposed optimal sharing rule without rational altruism

We start with a Cobb–Douglas production function (Cobb and Douglas, 1928) with decreasing returns to scale, as this function is widely accepted in economic literature. The use of a Cobb–Douglas production function is adequate for this study because it satisfies the conditions for ''team production'' as defined by Alchian and Demsetz (1972); it is capable of handling multiple inputs (in our study each input corresponds to the resource provided by a team member: a_k where $k = 1, \ldots, N$); and it exhibits strict complementarities among individual efforts. Moreover, it allows us to identify the elasticities (α_k) of the respective efforts (resources) that each team member (a_k) contributes to the team's output. Precisely, these elasticities will be the measure of the asymmetry of the team member's contribution to the team's output (or contribution-elasticity).

Thus, the Cobb–Douglas production function takes on the following form:

$$Y = F(a_1, \ldots, a_N) = \beta \prod_{k=1}^{N} a_k^{\alpha_k}; \quad 0 < \alpha_k < 1, \quad \sum_{k=1}^{N} \alpha_k < 1 \tag{1}$$

where Y is the total output from the vector of inputs and the given technology. The parameter β captures the level of total factor productivity and will be normalized to 1 for simplicity. The parameter a_k corresponds to the resource (effort) provided by team member k. The parameter α_k measures the elasticity of the output to changes in the quantity of resource provided by team member k (contribution-elasticity).[1] We assume that the elasticity varies across inputs and their values are positive and less than one.

The resource provided by team member i (a_i) has an external market so the opportunity cost of being used in the team production is given by $C_i(a_i) = \varpi_i \times a_i$ where ω_i is the market price of each unit of input.

Let S_i, such that $S_i > 0$ and $\sum_{k=1}^{N} S_k = 1$, be the share of total output assigned to team member i. The net utility of team member i corresponds to the true welfare (in the terminology of Rotemberg (1994) or material payoffs in terminology of Rabin (1993)). It is defined as $U_i = S_i \times Y - C_i(a_i)$, which for this particular case of the Cobb–Douglas production function and linear cost function, is equal to:

$$U_i = S_i \times \left(\prod_{k=1}^{N} a_k^{\alpha_k} \right) - \varpi_i \times a_i \tag{2}$$

Team member i will choose the quantity of input to supply to the joint production by maximizing U_i given in (2). The Nash equilibrium solution corresponding to the N first order conditions of the problems solved by the N members gives a solution of a_i as a function of the vector S_i, $i = 1$ to N; a_i (S_i, $S_{j \neq i}$) for $i = 1$ to N. For the given production and cost functions, the degree of effort of each team member will be (see Appendix):

$$a_i^* = \left(\frac{S_i \times \alpha_i}{\varpi_i} \right)^{1 - \sum_{j \neq i} \alpha_j / 1 - \sum_{k=1}^{N} \alpha_k} \prod_{j \neq i} \left(\frac{S_j \times \alpha_j}{\varpi_j} \right)^{\alpha_j / 1 - \sum_{k=1}^{N} \alpha_k} \tag{3}$$

The second best sharing rules are obtained from the solution to the optimization problem:

$$\underset{S_i}{Max} \left[F(a_i(S_i, S_{j \neq i}), a_{j \neq i}(S_{j \neq i}, S_{i \neq j})) - \sum_{k=1}^{N} (\varpi_k \times a_k(S_k, S_{m \neq k})) \right] \tag{4}$$

constrained to $R_i(Y) = (S_i \times Y) \geq C_i(a_i)$, $\forall i$ with $\sum_{k=1}^{N} S_k = 1$

[1] The α parameters are common knowledge to all team members.

For the particular production and cost functions and when the participation constraint $R_i(Y) = (S_i \times Y) \geq C_i(a_i)$, $\forall\, i$ is non-binding, the second best sharing rule for team member i, obtained from the solution of this problem, is given by (see Appendix):

$$S_i^* = \frac{\alpha_i}{\sum_{k=1}^{N} \alpha_k} \tag{5}$$

Result 1. *The second best sharing rule implies that team members with higher marginal productivities should receive a larger share of the output than team members with lower marginal productivities. If all team members have the same marginal productivities, then the second best sharing rule is an egalitarian sharing rule.*

2.2. A proposed optimal sharing rule with rational altruism

According to Rotemberg (1994), team members may have an incentive to show altruistic feelings (rational altruism; RA) even though their true interests are merely selfish preferences. Specifically, the concept of RA assumes that individuals are able to choose an optimal degree of altruism before production starts even though they are purely selfish.[2] By choosing altruism, individuals maximize their payoffs by anticipating how others will best respond to their revealed preferences. As in Rotemberg (1994), team members can credibly commit to their rationally chosen altruism even if ex post (the effort exertion phase) they desire not to act altruistically.

Specifically, under team production conditions, a selfish team member (i) will show rational altruism toward other team members ($j \neq i$) if he/she transmits the conviction to the other team members that his/her utility (U_i^{RA}) is:

$$U_i^{RA} = U_i(a_i, a_{j \neq i}) + \lambda_i \times \sum_{j \neq i} (U_j(a_j, a_{i \neq j})) \tag{6}$$

where $U_i(a_i,\, a_{j \neq i})$ represents the utility of team member i according to his/her effort (a_i) and that of other team members ($a_{j \neq i}$); $U_j(a_j, a_{i \neq j})$ represents the utility of team member j according to his/her effort a_j and that of other team members ($a_{i \neq j}$); and parameter λ_i represents how team member i shows that his/her utility is affected by that of team members $j \neq i$.[3] Under RA the value of λ_i is chosen strategically by the team members. If the λ_i parameter chosen is zero, then team member i behaves as though he/she were selfish; if the λ_i parameter chosen has a value higher than zero, then team member i behaves as though he/she were solidary.

When team member i can enhance his/her material payoffs $U_i(a_i,\, a_{j \neq i})$ by choosing $\hat{\lambda}_i > 0$ instead of $\hat{\lambda}_i = 0$, then parameter $\hat{\lambda}_i$ represents rational altruism. Consequently, when $\hat{\lambda}_i > 0$ team member i will act toward the rest of team members as if he/she were solidary, even though he/she is actually selfish.

When efforts are complementary, Rotemberg (1994) shows that rational altruism implies greater effort and greater welfare than would be expected from simple egoistical behavior. Therefore, rational altruism would bring team outcomes closer to first best outcomes compared to a scenario where rational altruism was absent.

In this section, we extend the team production problem to solve for the optimal sharing rule under rational altruism. In this scenario, in line with Rotemberg (1994, p. 690), once all team members have agreed on sharing rules, we also assume that each team member (i) chooses his/her rational altruism parameter ($\hat{\lambda}_i$) maximizing his/her material payoffs – *rational altruism subgame*. The chosen rational altruism parameters determine individual effort (a_i^{RA*}) – *effort exertion subgame*; and both (individual effort and rational altruism parameters) allow us to find the optimal sharing rules.

Therefore, we start with the maximization of each team member's individual utility (U_i^{RA}), with a sharing rule based on team output (S_i^{RA}) and assuming the presence of rational altruism.

$$\begin{aligned} \underset{a_i}{Max} U_i^{RA} &= \underset{a_i}{Max} \left[U_i(a_i, a_{j \neq i}) + \lambda_i \times \sum_{j \neq i} [U_j(a_j, a_{i \neq j})] \right] \\ &= \underset{a_i}{Max} \left[\left(S_i^{RA} \times \left(\prod_{k=1}^{N} a_k^{\alpha_k} \right) - \varpi_i \times a_i \right) + \lambda_i \times \sum_{j \neq i} \left[S_j^{RA} \times \left(\prod_{k=1}^{N} a_k^{\alpha_k} \right) - \varpi_j \times a_j \right] \right] \end{aligned} \tag{7}$$

[2] We need to assume that team members are purely selfish in order to isolate the rational altruism from other exogenous altruism components.

[3] We have to point out that in order to simplify the model we assume that each team member ''feels'' the same level of altruism (same rational altruism parameter: λ_i) for all the other team members. Although this assumption could reduce the applicability of the model, it does not affect our main objective which is the identification of the PERA.

The solution to this stage shows that the degree of effort of each team member (a_i^{RA*}) for given values of rational altruism parameters (λ_i) and given values of sharing rules (S_i^{RA}) is (see Appendix):

$$a_i^{RA*} = \left(\frac{\left(S_i^{RA} + \left(\sum_{j \neq i} S_j^{RA}\right) \times \lambda_i\right) \times \alpha_i}{\varpi_i} \right)^{1-\sum_{j \neq i}\alpha_j / \sum_{k=1}^{N}\alpha_k} \prod_{j \neq i} \left(\frac{\left(S_j^{RA} + \left(\sum_{i \neq j} S_i^{RA}\right) \times \lambda_j\right) \times \alpha_j}{\varpi_j} \right)^{\alpha_j / 1 - \sum_{k=1}^{N}\alpha_k} \quad (8)$$

Subsequently, each team member calculates the rational altruism value which maximizes his/her material payoffs, given effort $a_i^{RA^*}$ (8):

$$\underset{\lambda_i}{Max} U_i = S_i^{RA} \times \left(\prod_{k=1}^{N} (a_k^{RA^*})^{\alpha_k} \right) - \varpi_i \times a_i^{RA^*} \quad (9)$$

The degree of rational altruism shown by each team member ($\hat{\lambda}_i$) is thus (see Appendix):

$$\hat{\lambda}_i = \frac{S_i^{RA} \times \sum_{j \neq i}\alpha_j / 1 - \sum_{j \neq i}\alpha_j}{\sum_{j \neq i} S_j^{RA}} \quad (10)$$

Finally, we derive the problem of finding the optimal sharing rule $S_i^{RA^*}$, maximizing the wealth creation by the team, taking into account Eqs. (10) and (8):

$$\underset{S_i}{Max} \left[F(a_i^{RA*}(S_i^{RA}, S_{j \neq i}^{RA}), a_{j \neq i}^{RA*}(S_{j \neq i}^{RA}, S_{i \neq j}^{RA})) - \sum_{k=1}^{N} (\varpi_k \times a_k^{RA*}(S_k^{RA}, S_{m \neq k}^{RA})) \right] \quad (11)$$

constrained to $S_i^{RA} \times F(a_1^{RA*}, \ldots, a_n^{RA*}) \geq C_i(a_i^{RA*})$
with $\sum_{k=1}^{N} S_k^{RA} = 1$

Thus, the optimal sharing rule ($S_i^{RA^*}$) is (see Appendix):

$$S_i^{RA*} = \alpha_i / \sum_{k=1}^{N} \alpha_k \quad (12)$$

Result 2. *The second best sharing rule under rational altruism is the same as that without rational altruism ($S_i^* = S_i^{RA*}$).*

Substituting (12) in (10) the equilibrium value of the altruism parameter is

$$\hat{\lambda}_i^* = \frac{\alpha_i}{1 - \sum_{j \neq i}\alpha_j} = \frac{\alpha_i}{\alpha_i + \gamma} \quad (13)$$

where $\gamma = 1 - \left(\sum_k \alpha_k\right)$ is the degree of scale diseconomies in the production function.

Result 3. *The equilibrium rational altruism parameter is between zero and one. The more productive team members will show higher rational altruism than less productive ones. The equilibrium rational altruism parameter is lower in production technologies with higher scale diseconomies.*

The first part of the result comes from $\left(\alpha_i > 0, \forall i; \quad \text{and} \quad \alpha_i < 1 - \sum_{j \neq i}\alpha_j \quad \forall i\right)$. The second and third parts come from Eq. (13). This result is in accordance with Rotemberg (1994) which shows that the rational altruism parameter ranges from 0 to 1 ($0 < \hat{\lambda}_i^* < 1$).

The previous analysis shows how every team member becomes altruistic in his/her own self-interest causing an increase of his/her effort and, as a consequence, an improvement in the firm's efficiency.

3. Identification and assessment of perverse effects of rational altruism

The main contribution of the present paper is the identification and assessment of the perverse effects of rational altruism which are called the Perverse Element of Rational Altruism (PERA). The presence of the PERA is clearly detectable when at least one team member shows a rational altruism parameter higher than one. This generates an adverse value effect on wealth creation by the team. Specifically, from expression (6)[4] it can be deduced that the higher the RA parameter is over one, the greater the divergence between the team member's interest and the collective interest.

As noted earlier, when team member abilities are asymmetric, the second best optimal sharing rule assigns a different proportion of the output to each team member. As a consequence, the more productive team members receive a larger share of the output than the less productive ones. However, recent research argues in favor of the benefits of ''equal pay'' policies on teams (Bose et al., 2010; Bartling and Von Siemens, 2010). In this section we are interested in examining the implications of implementing equal sharing rules, other than the second best optimal one, on rational altruism behavior.

If the egalitarian sharing rule is chosen, $(1/N)$, then the equilibrium expression for the rational altruism parameter (from expression (10)) will be:

$$\hat{\lambda}_i \left(S_i^{RA} = \frac{1}{N} \right) = \frac{\sum_{j \neq i} \alpha_j / 1 - \sum_{j \neq i} \alpha_j}{(N-1)} \tag{14}$$

Positive elasticity of the effort of each of the team members (α_i) and decreasing returns to scale $\left(1 - \sum_{j \neq i} \alpha_j > 0\right)$ imply that equilibrium altruism parameters will all be positive $(\hat{\lambda}_i > 0)$ but different among team members. Moreover, Eq. (14) implies that the equilibrium altruism parameter for team member *i* is independent of his/her contribution-elasticity to the output (α_i).

Result 4. *Under an egalitarian sharing rule the equilibrium rational altruism parameter of team member i is independent of his/her contribution-elasticity to the output of the team (α_i). The equilibrium rational altruism parameter of member i increases with the average contribution-elasticity to the output of his/her teammates and decreases with the number of team members.*

Moreover, it is obvious that $\hat{\lambda}_i^*$ and $\hat{\lambda}_i$ will be equal if the team technology is symmetric and each member contributes equally to the output of the team $(\alpha_i = \alpha_j \; \forall i, j)$. It is of interest to examine the distortion in the choice of the rational altruism parameter induced by imposing an equal sharing rule, compared with the value of the parameter chosen under the second best sharing rule.

$$\hat{\lambda}_i^{PERA} = \hat{\lambda}_i - \hat{\lambda}_i^* = \frac{\sum_{j \neq i} \alpha_j / 1 - \sum_{j \neq i} \alpha_j}{N-1} - \frac{\alpha_i}{1 - \sum_{j \neq i} \alpha_j} = \frac{1}{N-1} \times \frac{\sum_{j \neq i} \alpha_j [(N-1) \times \alpha_i]}{1 - \sum_{j \neq i} \alpha_j} \tag{15}$$

Result 5. *An equal sharing rule implies a rational altruism parameter in equilibrium higher (lower) than the parameter chosen under the second best sharing rule for those team members with lower (higher) marginal productivities.*

We refer to the induced change in the rational altruism parameter from the parameter under the optimal second best sharing rule to the one under the equal sharing rule as the Perverse Element of Rational Altruism (PERA). The introduction of an equal sharing rule distorts the rational altruism parameter in such a way that those team members with lower marginal productivities now appear to be more altruistic than they were under the second best sharing rule and the opposite occurs with the more productive team members.

In Rotemberg (1994), the assumption that all team members are symmetric in terms of contribution-elasticity to team output and that the egalitarian sharing rule is the one chosen $(1/N)$ implies that the equilibrium solution will not be affected by the PERA. In the symmetric case all team members would have the same rational altruism parameters. Therefore, all team members would increase their levels of effort equally. Consequently all team members would benefit equally from the presence of rational altruism.

However, when team members are not homogeneous and the egalitarian sharing rule is the one chosen $(1/N)$, the less productive team members tend to exaggerate their altruism (positive PERA). These team members try to foster reciprocal rational altruism feelings from the most productive team members. So, less productive team members would benefit from the higher marginal productivities of their more productive teammates.

[4] Transforming expression (6) and expressing it in an equivalent manner: $U_i^{RA} = U_i(a_i, a_{j \neq i}) + \lambda_i \times \sum_{j \neq i} \left(U_j(a_j, a_{i \neq j}) \right) =$ (*If we apply this to our model*) $= \left[\frac{1}{N} \times \prod_{k=1}^{N} a_k^{\alpha_k} - (\varpi_i \times a_i) \right] + \lambda_i \times \sum_{j \neq i} \left[\frac{1}{N} \times \prod_{k=1}^{N} a_k^{\alpha_k} - \left(\varpi_j \times a_j \right) \right] = (1 - \lambda_i) \times \left[\frac{1}{N} \times \prod_{k=1}^{N} a_k^{\alpha_k} - [\varpi_i \times a_i] \right] + \lambda_i \times \left[\prod_{k=1}^{N} a_k^{\alpha_k} - \sum_{k=1}^{N} [\varpi_k \times a_k] \right].$

Table 1 Wealth creation.

	Without RA		With RA
Second-Best Sharing Rule	$WC^*_{S_i=S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}$	$\xrightarrow[(3^*)]{\Delta WC}$	$WC^{*RA}_{S_i=S_i^*=\frac{\alpha_i}{\sum_{K=1}^{N}\alpha_k}}$
	$\Delta WC \uparrow$ (1*)		(2*) $\uparrow \Delta WC$
Egalitarian Sharing Rule	$WC^*_{S_i=\frac{1}{N}}$	$\xrightarrow[\Delta WC]{(4^*)}$	$WC^{*RA}_{S_i=\frac{1}{N}}$

However, the more productive team members anticipate that the less productive team members will increase their parameters of altruism and, therefore, they tend to decrease their levels of altruism. In terms of wealth creation, the negative effect derived from lower effort levels of the more productive team members is greater than the positive effect derived from the higher effort levels of the less productive team members. Therefore, the presence of the PERA decreases team efficiency.

4. Welfare considerations

In this section we examine the changes in total wealth creation of moving from an equal sharing to an optimal second best sharing rule in heterogeneous teams while also allowing for the possibility of rational altruism of team members. The situations to be compared are summarized in Table 1 where the rows represent the two different sharing rules and the two columns differentiate between teams where members practice rational altruism and teams that do not.

All changes in welfare are positive, therefore by definition moving from an equal sharing rule to a second best sharing rule increases total wealth creation [relations (1*) and (2*)]. Also, rational altruism improves wealth creation in teams that apply equal sharing rules and in teams that apply second best sharing rules since rational altruism parameters for all team members are always positive [relations (3*) and (4*)].

In the case of no altruism and asymmetric teams, the use of egalitarian sharing rules (or moving away from the second best sharing rule) decreases team efficiency with respect to the second best sharing rule by lowering the effort levels of the most productive team members relative to the levels under the second best sharing rule. When the sharing rule gives a larger part of the output to the more productive members (using the second best sharing rule) they increase their efforts and team efficiency is improved even though less productive team members decrease their effort levels.

If the team members engage in some form of rational altruism, then the equal sharing rule also distorts the equilibrium altruism parameters such that less productive team members tend to over play their altruism at the expense of the more productive ones. When the sharing rule gives a larger part of the output to the more productive members then the gain through rational altruism by the less productive team members is lower and so they refrain from showing their PERA levels, increasing RA levels of the most productive team members and thereby improving team efficiency.

We are interested in examining when the increases in welfare are higher. We conjecture that the opportunity cost, in terms of lost welfare, of implementing equal sharing rules in asymmetric teams is higher when the members of the team engage in some form of rational altruism.

Specifically, when the team is composed of two members ($N=2$), the following relationship holds for wealth creation by the two team members (see Appendix) in the four situations shown in Table 1 (see Appendix):

$$\frac{WC^*_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;\ S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}}{WC^*_{S_1=S_2=1/2}}=\frac{WC^{*RA}_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;\ S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}}{WC^{*RA}_{S_1=S_2=1/2}} \tag{16}$$

From expression (16) and taking into account the results shown in Table 1 ($WC^{*RA}_{S_1=S_2=1/2}>WC^*_{S_1=S_2=1/2}$; $WC^{*RA}_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;\ S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}>WC^*_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;\ S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}$ and $WC^*_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}>WC^*_{S_1=S_2=1/2}$) we obtain that ($WC^{*RA}_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;\ S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}-WC^{*RA}_{S_1=S_2=1/2}$) $>$ ($WC^*_{S_1=S_1^*=\alpha_1/\alpha_1+\alpha_2;\ S_2=S_2^*=\alpha_2/\alpha_1+\alpha_2}-WC^*_{S_1=S_2=1/2}$). Therefore, we can conclude that the application of second best sharing rules improves wealth creation more than the application of equal sharing rules, and with rational altruism more than without rational altruism.

This means that the relative gain in welfare of going from equal sharing to second best sharing rules is the same with rational altruism as without altruism. Since the initial welfare base is lower in the case of no altruism, the absolute increase in welfare must be higher when the shift from equal to second best sharing rules occurs in teams with rational altruism. In other words, the opportunity cost, in terms of lost welfare, of implementing equal sharing rules in heterogeneous teams is higher when the members of the team engage in some form of rational altruism. The PERA probably explains this result. Departing from the second best sharing rule in these teams creates an additional distortion of team member behavior such that those with low contribution-elasticity to the team's output overplay their altruism in the hope of increasing the effort of the more productive team members and thereby obtaining additional benefits thanks to the equal sharing rule. When the sharing rule gives a larger part of the output to the more productive members then the gain from rational altruism by the less productive team members is lower and so they refrain from showing their PERA levels. These conclusions are symmetric so we can argue that introducing rational altruism in a team that initially does not practice it will provide a larger increase in total wealth creation when the team uses second best sharing rules than when it uses equal sharing rules.

5. Concluding remarks

There is currently no consensus on the appropriateness of using egalitarian sharing rules. Specialized literature gives arguments both in favor of, and against, the use of this rule in different scenarios. Therefore, our paper elaborates on this current debate and provides new arguments against the application of the egalitarian sharing rules when team members have different marginal productivities.

Specifically, when team members display some form of rational altruism (Rotemberg, 1994), we find that the egalitarian sharing rule modifies the rational altruism parameter of team members compared to the parameter chosen in the case of the second best sharing rule (the sharing rule based on relative marginal productivities). In this paper this distortion of the rational altruism parameter is called the *Perverse Element of Rational Altruism (PERA)*. It means that, under equal sharing rules and heterogeneous team members, the less productive team members tend to exaggerate their altruism with the expectation of inducing increased effort from the more productive team members and benefiting from it. On the contrary, the more productive team members anticipate that the less productive ones will increase their parameters of altruism and, therefore, they tend to reduce their altruism. In this scenario rational altruism continues to improve wealth creation in teams with equal sharing rules, compared to the wealth creation in a no altruism scenario, but to a lesser degree than it would under second best sharing rules. Moreover, the opportunity cost, in terms of lost welfare, of implementing equal sharing rules in heterogeneous teams is higher in teams that engage in rational altruism than in teams that do not.

All these conclusions lose relevance in homogeneous teams since in this case the second best sharing rule is the egalitarian one, so all potential distortions disappear. If the recommendation of forming homogeneous teams holds whenever there are restrictions to applying sharing rules other than the egalitarian sharing rule, it is even more applicable in cases where team members can be expected to show some form of rational altruism.

The paper could be extended to other production functions different from the Cobb–Douglas one but we expect that the qualitative conclusions of our results would be the same. Another important line of future research would be to identify the existence of the PERA in experiments; rational altruism is difficult to isolate in empirical observation of behavior so we are unaware of how important it can be in actual team production activities. Controlled experiments should help to isolate this behavior as well as the effect of modifying the sharing rules with and without rational altruism.

Appendix.

A.1. Demonstration Eq. (3)

Eq. (2) is:

$$U_i = S_i\left(\prod_{k=1}^{N}\alpha_k^{\alpha_k}\right) - \omega_i\alpha_i$$

With two team members:

$$u_1 = S_1 a_1^{\alpha_1} a_2^{\alpha_2} - \omega_1 a_1$$
$$u_2 = S_2 a_2^{\alpha_1} a_2^{\alpha_2} - \omega_2 a_2$$

First order conditions:

$$\frac{\partial U_1}{\partial a_1}=0\Rightarrow a_1=\left(\frac{\omega_1}{S_1\alpha_1 a_2^{\alpha_2}}\right)^{1/\alpha_1-1} \qquad \frac{\partial U_2}{\partial a_2}=0\Rightarrow a_2=\left(\frac{\omega_2}{S_2\alpha_2 a_1^{\alpha_1}}\right)^{1/\alpha_2-1}$$

$$a_1=\left(\frac{\omega_1}{S_1\alpha_1(\omega_2/S_2\alpha_2 a_1^{\alpha_1})^{\alpha_2/\alpha_2-1}}\right)^{1/\alpha_1-1} \qquad a_2=\left(\frac{\omega_2}{S_2\alpha_2(\omega_1/S_1\alpha_1 a_2^{\alpha_2})^{\alpha_1/\alpha_1-1}}\right)^{1/\alpha_2-1}$$

$$a_1^*=\left(\frac{S_1\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{S_2\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2} \qquad a_2^*=\left(\frac{S_2\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{S_1\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}$$

With N team members:

$$a_i^*=\left(\frac{S_i\alpha_i}{\omega_i}\right)^{\frac{1-\sum_{j\neq i}\alpha_j}{1-\sum_{k=1}^{N}\alpha_k}}\prod_{j\neq i}\left(\frac{S_j\alpha_j}{\omega_j}\right)^{\frac{a_j}{1-\sum_{k=1}^{N}\alpha_k}}$$

A.2. Demonstration of Eq. 5

Eq. (4) is:

$$\underset{S_i}{Max}\left[F(a_i(S_i,S_{j\neq i}),a_{j\neq i}(S_{j\neq i},S_{i\neq j}))-\sum_{i=1}^{N}(\varpi_i\times a_i(S_i,S_{j\neq i}))\right]$$

With two team members:

$$\max_{S_i}(a_1^{*\alpha_1}a_2^{*\alpha_2}-\omega_1 a_1^*-\omega_2 a_1^*)$$

$$a_1^*=\left(\frac{S_1\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{S_2\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2} \qquad a_2^*=\left(\frac{S_2\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{S_1\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}$$

$$\max_{S_i}\left(\left(\left(\frac{S_1\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{S_2\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\right)^{\alpha_1}\left(\left(\frac{S_2\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2}\right)\left(\frac{S_1\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\right)^{\alpha^2}$$
$$-\,\omega_1\left(\left(\frac{S_1\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{S_2\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\right)-\omega_2\left(\left(\frac{S_2\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{S_1\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\right)$$

$$\max_{S_i}\left(S_1^{\alpha_1/1-\alpha_1-\alpha_2}S_2^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\left[1-\omega_1\left(\frac{\alpha_1}{\omega_1}\right)S_1-\omega_2\left(\frac{\alpha_2}{\omega_2}\right)S_2\right]\right)$$

$$S_2=1-S_1$$

$$\max_{S_i}\left(S_1^{\alpha_1/1-\alpha_1-\alpha_2}(1-S_1)^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}[1-\alpha_1 S_1-\alpha_2+\alpha_2 S_1]\right)$$

$$\left[S_1^{\alpha_1/1-\alpha_1-\alpha_2}(1-S_1)^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}[1-\alpha_1 S_1-\alpha_2+\alpha_2 S_1]\right]=RN$$

First order conditions:

$$\frac{\partial RN}{\partial S_1} = RN \quad \frac{\partial(\ln RN)}{\partial S_1} = 0$$

$$RN\frac{\partial(\ln RN)}{\partial S_1} = \left[S_1^{\alpha_1/1-\alpha_1-\alpha_2}(1-S_1)^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}[1-\alpha_1 S_1-\alpha_2+\alpha_2 S_1]\right]$$
$$\times\left[\frac{\alpha_1}{1-\alpha_1-\alpha_2}\frac{1}{S_1}+\frac{\alpha_2}{1-\alpha_1-\alpha_2}\frac{-1}{1-S_1}+\frac{a_2-a_1}{1-\alpha_1 S_1-\alpha_2+\alpha_2 S_1}\right] = 0$$

$$\alpha_1 = S_1(\alpha_1+\alpha_2)$$

$$S_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} \quad S_2 = \frac{\alpha_2}{\alpha_1+\alpha_2} \quad S_1+S_2 = 1$$

With N team members:

$$S_i = \frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}$$

A.3. Demonstration of Eq. (8)

Eq. (7) is:

$$\underset{a_i}{Max}U_i^{RA} = \underset{a_i}{Max}\left[U_i(a_i, a_{j\neq i}) + \lambda_i \times \sum_{j\neq i}[U_j(a_j, a_{i\neq j})]\right]$$
$$= \underset{a_i}{Max}\left[\left(S_i^{RA}\times\left(\prod_{k=1}^{N}a_k^{\alpha_k}\right)-\varpi_i\times a_i\right)+\lambda_i\times\sum_{j\neq i}\left[S_j^{RA}\times\left(\prod_{k=1}^{N}a_k^{\alpha_k}\right)-\varpi_j\times a_j\right]\right]$$

With two team members:

$$U_1^{RA} = S_1^{RA}a_1^{\alpha_1}a_2^{\alpha_2}-\omega_1 a_1+\lambda_1(S_2^{RA}a_1^{\alpha_1}a_2^{\alpha_2}-\omega_2 a_2)$$
$$U_2^{RA} = S_2^{RA}a_1^{\alpha_1}a_2^{\alpha_2}-\omega_2 a_2+\lambda_2(S_1^{RA}a_1^{\alpha_1}a_2^{\alpha_2}-\omega_1 a_1)$$

First order conditions:

$$\frac{\partial U_1^{RA}}{\partial a_1} = 0 \Rightarrow a_1 = \left(\frac{\omega_1}{(S_1^{RA}+S_2^{RA}\lambda_1)\alpha_1 a_2^{\alpha_2}}\right)^{1/\alpha_1-1}$$

$$\frac{\partial U_2^{RA}}{\partial a_2} = 0 \Rightarrow a_2 = \left(\frac{\omega_2}{(S_2^{RA}+S_1^{RA}\lambda_2)\alpha_2 a_1^{\alpha_1}}\right)^{1/\alpha_2-1}$$

$$a_1 = \left(\frac{\omega_1}{(S_1^{RA}+S_2^{RA}\lambda_1)\alpha_1(\omega_2/(S_2^{RA}+S_1^{RA}\lambda_2)\alpha_2 a_1^{\alpha_1})^{\alpha_2/\alpha_2-1}}\right)^{1/\alpha_1-1}$$

$$a_2 = \left(\frac{\omega_2}{(S_2^{RA}+S_1^{RA}\lambda_2)\alpha_2(\omega_1/(S_1^{RA}+S_2^{RA}\lambda_1)\alpha_1 a_2^{\alpha_2})^{\alpha_1/\alpha_1-1}}\right)^{1/\alpha_2-1}$$

$$a_1^{RA*} = \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2} \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}$$

$$a_2^{RA*} = \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2} \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}$$

With N team members:

$$a_i^{RA*} = \left(\frac{\left(S_i^{RA} + \left(\sum_{j\neq i} S_j^{RA}\right) \times \lambda_i\right) \times \alpha_i}{\varpi_i}\right)^{1-\sum_{j\neq i}\alpha_j/1-\sum_{k=1}^{N}\alpha_k} \prod_{j\neq i}\left(\frac{\left(S_j^{RA} + \left(\sum_{i\neq j} S_i^{RA}\right) \times \lambda_j\right) \times \alpha_j}{\varpi_j}\right)^{\alpha_j/\sum_{k=1}^{N}\alpha_k}$$

A.4. Demonstration of Eq. (10)

Eq. (9) is:

$$\underset{\lambda_i}{Max} U_i = S_i^{RA} \times \left(\prod_{k=1}^{N} (a_k^{RA*})^{\alpha_k}\right) - \varpi_i \times a_i^{RA*}$$

With two team members:

$$U_2 = S_2^{RA} a_1^{*RA^{\alpha_1}} a_2^{*RA^{\alpha_2}} - \omega_2 a_2^{RA*} \quad U_1 = S_1^{RA} a_1^{*RA^{\alpha_1}} a_2^{*RA^{\alpha_2}} - \omega_1 a_1^{RA*}$$

$$a_1^{RA*} = \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2} \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}$$

$$a_2^{RA*} = \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2} \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}$$

$$\begin{aligned} U_1 = S_1^{RA}&\left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2} \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2} \\ &- \omega_1\left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2} \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2} \end{aligned}$$

$$\begin{aligned} U_2 = S_2^{RA}&\left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2} \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2} \\ &- \omega_2\left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2} \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2} \end{aligned}$$

First order conditions:

$$\frac{\partial U_1}{\partial \lambda_1} = 0 \Rightarrow \lambda_1 = \frac{(S_1^{RA}\alpha_2/1-\alpha_2)}{S_2^{RA}} \quad \frac{\partial U_2}{\partial \lambda_2} = 0 \Rightarrow \lambda_2 = \frac{S_2^{RA}\alpha_1/1-\alpha_1}{S_1^{RA}}$$

With N team members:

$$\hat{\lambda}_i = \frac{S_i^{RA} \times \left(\sum_{j \neq i}\alpha_j/1 - \sum_{j \neq i}\alpha_j\right)}{\sum_{j \neq i} S_j^{RA}}$$

A.5. Demonstration of Eq. (12)

Eq. (11) is:

$$\underset{S_i^{RA}}{Max}\left[F(a_i^{RA*}(S_i^{RA}, S_{j\neq i}^{RA}), a_{j\neq i}^{RA*}(S_{j\neq i}^{RA}, S_{i\neq j}^{RA})) - \sum_{i=1}^{N}(\varpi_i \times a_i^{RA*}(S_i^{RA}, S_{j\neq i}^{RA}))\right]$$

With two team members:

$$\max_{S_i^{RA}}(a_1^{*RA^{\alpha_1}} a_2^{*RA^{\alpha_2}} - \omega_1 a_1^{*RA} - \omega_2 a_2^{*RA})$$

$$a_1^{RA*} = \left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{1-\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}$$

$$a_2^{RA*} = \left(\frac{(S_2^{RA} + S_1^{RA}\lambda_2)\alpha_2}{\omega_2}\right)^{1-\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{(S_1^{RA} + S_2^{RA}\lambda_1)\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}$$

$$\lambda_1 = \frac{S_1^{RA}\alpha_2/1-\alpha_2}{S_2^{RA}} \quad \lambda_2 = \frac{S_2^{RA}\alpha_1/1-\alpha_1}{S_1^{RA}}$$

$$\max_{S_i^{RA}}\left(\left(\frac{S_1^{RA}}{1-\alpha_2}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{S_2^{RA}}{1-\alpha_1}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\left[1 - S_1^{RA}\frac{\alpha_1}{1-\alpha_2} - S_2^{RA}\frac{\alpha_2}{1-\alpha_1}\right]\right)$$

$$S_2 = 1 - S_1$$

$$\max_{S_1}\left(S_1^{RA(\alpha_1/1-\alpha_1-\alpha_2)}(1-S_1^{RA})^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{1}{1-\alpha_2}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{1}{1-\alpha_1}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\right.$$
$$\left.\times\left[1 - S_1^{RA}\frac{\alpha_1}{1-\alpha_2} - \frac{\alpha_2}{1-\alpha_1} + S_1^{RA}\frac{\alpha_2}{1-\alpha_1}\right]\right)$$

$$\left[S_1^{RA(\alpha_1/1-\alpha_1-\alpha_2)}(1-S_1^{RA})^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{1}{1-\alpha_2}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{1}{1-\alpha_1}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\omega_1}\right)^{\alpha_1/1-\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\omega_2}\right)^{\alpha_2/1-\alpha_1-\alpha_2}\right.$$
$$\left.\times\left[1 - S_1^{RA}\frac{\alpha_1}{1-\alpha_2} - \frac{\alpha_2}{1-\alpha_1} + S_1^{RA}\frac{\alpha_2}{1-\alpha_1}\right]\right] = RN$$

First order conditions:

$$\frac{\partial RN}{\partial S_1} = RN\frac{\partial(\ln RN)}{\partial S_1} = 0$$

$$RN\frac{\partial(\ln RN)}{\partial S_1} = 0$$

$$\alpha_1 = S_1^{RA}(\alpha_1 + \alpha_2)$$

$$S_1^{RA} = \frac{\alpha_1}{\alpha_1 + \alpha_2} \quad S_2^{RA} = \frac{\alpha_2}{\alpha_1 + \alpha_2} \quad S_1^{RA} + S_2^{RA} = 1$$

With N team members:

$$S_i^{RA*} = \frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}$$

A.6. Demonstration of Eq. (16)

$$\frac{WC^*_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}}{WC^*_{S_i=\frac{1}{N}}} = \frac{WC^{*RA}_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}}{WC^{*RA}_{S_i=\frac{1}{N}}} \Leftrightarrow \frac{\left(WC^*_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}\right)\times\left(WC^{*RA}_{S_i=\frac{1}{N}}\right)}{\left(WC^{*RA}_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}\right)\times\left(WC^*_{S_i=\frac{1}{N}}\right)} = 1$$

$$\frac{\left(WC^*_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}\right)\times\left(WC^{*RA}_{S_i=\frac{1}{N}}\right)}{\left(WC^{*RA}_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}\right)\times\left(WC^*_{S_i=\frac{1}{N}}\right)}$$

$$
= \frac{
\left(\prod_{k=1}^{N}\left[\left(\frac{\alpha_k}{\omega_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \prod_{k=1}^{N}\left[\left(\frac{\alpha_k}{\sum_{k=1}^{N}\alpha_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \left[1-\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\sum_{k=1}^{N}\alpha_k}\right]\right]\right)
\times \left(\prod_{k=1}^{N}\left[\left(\frac{\alpha_k}{\omega_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \left(\frac{1}{N}\right)^{\frac{\sum_{k=1}^{N}\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}} \times \prod_{k=1}^{N}\left[\left(\frac{1}{1-\alpha_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \left[1-\frac{1}{N}\times\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right]\right)
}{
\left(\prod_{k=1}^{N}\left[\left(\frac{\alpha_k}{\omega_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \prod_{k=1}^{N}\left[\left(\frac{\alpha_k}{\sum_{k=1}^{N}\alpha_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \prod_{k=1}^{N}\left[\left(\frac{1}{1-\alpha_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \left[1-\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(\sum_{k=1}^{N}\alpha_k\right)\times\left(1-\sum_{j\neq k}\alpha_j\right)}\right]\right]\right)
\times \left(\prod_{k=1}^{N}\left[\left(\frac{\alpha_k}{\omega_k}\right)^{\frac{\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}}\right] \times \left(\frac{1}{N}\right)^{\frac{\sum_{k=1}^{N}\alpha_k}{1-\sum_{k=1}^{N}\alpha_k}} \times \left[1-\frac{1}{N}\times\sum_{k=1}^{N}\alpha_k\right]\right)
}
$$

$$
= \frac{\left[1-\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\sum_{k=1}^{N}\alpha_k}\right]\right] \times \left[1-\frac{1}{N}\times\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right]}{\left[1-\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(\sum_{k=1}^{N}\alpha_k\right)\times\left(1-\sum_{j\neq k}\alpha_j\right)}\right]\right] \times \left[1-\frac{1}{N}\times\sum_{k=1}^{N}\alpha_k\right]}
$$

$$
= \frac{1-\left(\frac{1}{N}\times\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right)-\left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\sum_{k=1}^{N}\alpha_k}\right]\right)+\left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\sum_{k=1}^{N}\alpha_k}\right]\times\frac{1}{N}\times\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right)}{1-\left(\frac{1}{N}\times\sum_{k=1}^{N}\alpha_k\right)-\left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(\sum_{k=1}^{N}\alpha_k\right)\times\left(1-\sum_{j\neq k}\alpha_j\right)}\right]\right)+\left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(\sum_{k=1}^{N}\alpha_k\right)\times\left(1-\sum_{j\neq k}\alpha_j\right)}\right]\times\frac{1}{N}\times\sum_{k=1}^{N}\alpha_k\right)}
$$

$$= \frac{1 - \left(\frac{1}{N} \times \sum_{k=1}^{N} \left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right] \times \frac{\sum_{k=1}^{N}\alpha_k}{\sum_{k=1}^{N}\alpha_k}\right) - \left(\frac{1}{\sum_{k=1}^{N}\alpha_k} \times \sum_{k=1}^{N}\left[\alpha_k^2\right] \times \frac{N}{N}\right) + \left(\frac{1}{\sum_{k=1}^{N}\alpha_k} \times \sum_{k=1}^{N}\left[\alpha_k^2\right] \times \frac{1}{N} \times \sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right)}{1 - \left(\frac{1}{N} \times \sum_{k=1}^{N}\alpha_k \times \frac{\sum_{k=1}^{N}\alpha_k}{\sum_{k=1}^{N}\alpha_k}\right) - \left(\frac{1}{\sum_{k=1}^{N}\alpha_k} \times \sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(1-\sum_{j\neq k}\alpha_j\right)}\right] \times \frac{N}{N}\right) + \left(\frac{1}{\sum_{k=1}^{N}\alpha_k} \times \sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(1-\sum_{j\neq k}\alpha_j\right)}\right] \times \frac{1}{N} \times \sum_{k=1}^{N}\alpha_k\right)}$$

$$= \frac{\dfrac{N\times\sum_{k=1}^{N}\alpha_k - \left(\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right] \times \sum_{k=1}^{N}\alpha_k\right) - \left(\sum_{k=1}^{N}\left[\alpha_k^2\right]\times N\right) + \left(\sum_{k=1}^{N}\left[\alpha_k^2\right]\times\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right)}{N\times\sum_{k=1}^{N}\alpha_k}}{\dfrac{N\times\sum_{k=1}^{N}\alpha_k - \left(\sum_{k=1}^{N}\alpha_k\times\sum_{k=1}^{N}\alpha_k\right) - \left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(1-\sum_{j\neq k}\alpha_j\right)}\right]\times N\right) + \left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(1-\sum_{j\neq k}\alpha_j\right)}\right]\times\sum_{k=1}^{N}\alpha_k\right)}{N\times\sum_{k=1}^{N}\alpha_k}}$$

$$= \frac{N \times \sum_{k=1}^{N}\alpha_k - \left(\sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right] \times \sum_{k=1}^{N}\alpha_k\right) - \left(\sum_{k=1}^{N}\left[\alpha_k^2\right] \times N\right) + \left(\sum_{k=1}^{N}\left[\alpha_k^2\right] \times \sum_{k=1}^{N}\left[\frac{\alpha_k}{1-\sum_{j\neq k}\alpha_j}\right]\right)}{N \times \sum_{k=1}^{N}\alpha_k - \left(\sum_{k=1}^{N}\alpha_k \times \sum_{k=1}^{N}\alpha_k\right) - \left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(1-\sum_{j\neq k}\alpha_j\right)}\right] \times N\right) + \left(\sum_{k=1}^{N}\left[\frac{\alpha_k^2}{\left(1-\sum_{j\neq k}\alpha_j\right)}\right] \times \sum_{k=1}^{N}\alpha_k\right)}$$

$$\text{If} \quad N = 2 \quad \text{then} \quad \frac{\left(WC^{*}_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}\right) \times \left(WC^{*RA}_{S_i=\frac{1}{N}}\right)}{\left(WC^{*RA}_{S_i^*=\frac{\alpha_i}{\sum_{k=1}^{N}\alpha_k}}\right) \times \left(WC^{*}_{S_i=\frac{1}{N}}\right)} = \frac{\left(WC^{*RA}_{S_1=S_2=\frac{1}{2}}\right) \times \left(WC^{*}_{S_1^*=\frac{\alpha_1}{\alpha_1+\alpha_2};\ S_2^*=\frac{\alpha_2}{\alpha_1+\alpha_2}}\right)}{\left(WC^{*RA}_{S_1^*=\frac{\alpha_1}{\alpha_1+\alpha_2};\ S_2^*=\frac{\alpha_2}{\alpha_1+\alpha_2}}\right) \times \left(WC^{*}_{S_1=S_2=\frac{1}{2}}\right)}$$

$$= \frac{2 \times (\alpha_1 + \alpha_2) - (\alpha_1 + \alpha_2) \times \left(\frac{\alpha_1}{1-\alpha_2} + \frac{\alpha_2}{1-\alpha_1}\right) - 2 \times \left(\alpha_1^2 + \alpha_2^2\right) + \left(\alpha_1^2 + \alpha_2^2\right) \times \left(\frac{\alpha_1}{1-\alpha_2} + \frac{\alpha_2}{1-\alpha_1}\right)}{2 \times (\alpha_1 + \alpha_2) - (\alpha_1 + \alpha_2)^2 - 2 \times \left(\frac{\alpha_1^2}{1-\alpha_2} + \frac{\alpha_2^2}{1-\alpha_1}\right) + (\alpha_1 + \alpha_2) \times \left(\frac{\alpha_1^2}{1-\alpha_2} + \frac{\alpha_2^2}{1-\alpha_1}\right)}$$

$$= \frac{\begin{array}{c} 2 \times (\alpha_1 + \alpha_2) \times (1-\alpha_1) \times (1-\alpha_2) - (\alpha_1 + \alpha_2) \times [\alpha_1 \times (1-\alpha_1) + \alpha_2 \times (1-\alpha_2)] - 2 \times \left(\alpha_1^2 + \alpha_2^2\right) \\ \times (1-\alpha_1) \times (1-\alpha_2) + \left(\alpha_1^2 + \alpha_2^2\right) \times [\alpha_1 \times (1-\alpha_1) + \alpha_2 \times (1-\alpha_2)] \end{array}}{\begin{array}{c} 2 \times (\alpha_1 + \alpha_2) \times (1-\alpha_1) \times (1-\alpha_2) - (\alpha_1 + \alpha_2)^2 \times (1-\alpha_1) \times (1-\alpha_2) - 2 \\ \times \left[\alpha_1^2 \times (1-\alpha_1) + \alpha_2^2 \times (1-\alpha_2)\right] + (\alpha_1 + \alpha_2) \times \left[\alpha_1^2 \times (1-\alpha_1) + \alpha_2^2 \times (1-\alpha_2)\right] \end{array}}$$

$$= \frac{\begin{array}{c} -\alpha_1^4 - \alpha_2^4 - 2 \times \alpha_1^3 \times \alpha_2 - 2 \times \alpha_2^3 \times \alpha_1 + 4 \times \alpha_1^3 + 4 \times \alpha_2^3 + 6 \times \alpha_1^2 \times \alpha_2 + 6 \times \alpha_2^2 \times \alpha_1 - 2 \times \alpha_1^2 \times \alpha_2^2 \\ -5 \times \alpha_1^2 - 5 \times \alpha_2^2 - 6 \times \alpha_1 \times \alpha_2 + 2 \times \alpha_1 + 2 \times \alpha_2 \end{array}}{\begin{array}{c} -\alpha_1^4 - \alpha_2^4 - 2 \times \alpha_1^3 \times \alpha_2 - 2 \times \alpha_2^3 \times \alpha_1 + 4 \times \alpha_1^3 + 4 \times \alpha_2^3 + 6 \times \alpha_1^2 \times \alpha_2 + 6 \times \alpha_2^2 \times \alpha_1 \\ -2 \times \alpha_1^2 \times \alpha_2^2 - 5 \times \alpha_1^2 - 5 \times \alpha_2^2 - 6 \times \alpha_1 \times \alpha_2 + 2 \times \alpha_1 + 2 \times \alpha_2 \end{array}} = 1$$

References

Alchian, A., Demsetz, H., 1972. Production, information costs and economic organization. The American Economic Review 62 (5), 777–795.

Bartling, B., Von Siemens, F.A., 2010. Equal sharing rules in partnerships. Journal of Institutional and Theoretical Economics 166 (2), 299–320.

Bose, A., Pal, D., Sappington, D.E.M., 2010. Equal pay for unequal work: limiting sabotage in teams. Journal of Economics and Management Strategy 19 (1), 25–53.

Cobb, C.W., Douglas, P.H., 1928. A theory of production. The American Economic Review XVIII (1), 139–165.

Farrell, J., Scotchmer, S., 1988. Partnerships. The Quarterly Journal of Economics 103 (2), 279–297.

Fehr, E., Schmidt, K.M., 1999. A theory of fairness, competition, and cooperation. The Quarterly Journal of Economics 114, 817–868.

Goerg, S.J., Kube, S., Zultan, R., 2010. Treating equals unequally: incentives in teams, workers' motivation, and production technology. Journal of Labor Economics 28 (4), 747–772.

Hamilton, B.H., Nickerson, J.A., Owan, H., 2003. Team incentives and worker heterogeneity: an empirical analysis of the impact of teams on productivity and participation. Journal of Political Economy 111 (3), 465–497.

Holmström, B., 1982. Moral hazard in teams. The Bell Journal of Economics 13 (2), 324–340.

Kräkel, M., Steiner, G., 2001. Equal sharing in partnerships? Economics Letters 73, 105–109.

Lazear, E.P., Shaw, K.L., 2007. Personnel economics: the economist's view of human resources. Journal of Economic Perspectives 21 (4), 91–114.

Rabin, M., 1993. Incorporating fairness into game theory and economics. The American Economic Review 83, 1281–1302.

Rotemberg, J.J., 1994. Human relations in the workplace. Journal of Political Economy 102 (4), 684–717.

Sherstyuk, K., 1998. Efficiency in partnership structures. Journal of Economic Behavior and Organization 36, 331–346.

Wilson, R., 1968. The theory of syndicates. Econometrica 36 (1), 119–132.